

Dips in Partial Wave Amplitudes from Final State Interactions

Charles B. Chiu and Duane A. Dicus

Abstract

We consider the dip-peak structures in the $J=0$ partial wave amplitudes for processes $\gamma\gamma \rightarrow W^+W^-$ and $\gamma\gamma, gg \rightarrow t\bar{t}$ taking into account the corresponding Born term process and the rescattering process where the intermediate state is rescattered through the exchange of Higgs resonance state in the direct channel.

Near a direct channel resonance, final state interactions can be important even for relatively weakly interacting particles. Consider the production by photons of a pair of W s, $\gamma\gamma \rightarrow W^+W^-$. The Born amplitude is of order e^2 and rescattering by the W s naively makes the amplitude of order e^4 . However, if the rescattering process corresponds to the exchange of, such as in this example, the Higgs in the direct channel, near the Higgs resonance the rescattering amplitude goes as $\frac{e^4 m}{\Gamma} \sim e^2$, where Γ is the resonance width, and m is the resonance mass.

A characteristic signature of a final state interaction near a resonance is a *dip* in the scattering amplitude.[1-7] This dip occurs for the part of the amplitude where the rescattering particles are on mass shell and when there is only a single channel involved.¹ In practice, the dip may be washed out by the part of the amplitude with the particles off-shell, other particles produced and rescattered into the desired final state (including other polarizations of the final state particles), and even contributions from other particles than the final state particles to Γ . Our goal in this note is to consider a few simple reactions in the Standard Model and check to what extent a dip will manifest itself in the partial wave amplitude. We choose to work directly with the $J=0$ partial wave amplitude, since the interference effect should show up most prominently in partial wave amplitudes.

As a definite, and uncomplicated example consider again $\gamma\gamma \rightarrow W^+W^-$. First take only the Goldstone boson part of the W s, $\gamma\gamma \rightarrow \chi^+\chi^-$ and work in the $\xi = 1$ gauge where $m_\chi = M_W$. The Born $J = 0$ amplitude is

$$a_{++}^{\text{Born}} = \frac{e^2}{16\pi} \frac{(1 - \beta^2)}{\beta} \ln \frac{1 + \beta}{1 - \beta} \quad (1)$$

where β is the χ velocity, $\beta = (1 - M_W^2/E^2)^{\frac{1}{2}}$, with E the initial photon energy.

The amplitude for two photons to produce an off-shell Higgs is

$$\begin{aligned} \epsilon_\mu(k_1)\epsilon_\nu(k_2)T^{\mu\nu} &= \frac{e^3 M_W \pi^2}{\sin \theta_W (2\pi)^4} \left(g^{\mu\nu} - \frac{2k_1^\nu k_2^\mu}{s} \right) \epsilon_\mu(k_1)\epsilon_\nu(k_2) \\ &\times \left[\frac{m_H^2}{M_W^2} + \frac{m_H^2}{s} I \right] \end{aligned} \quad (2)$$

¹See in particular the discussion in the Appendix of Ref. 5.

where $s = 4E^2$ and

$$I = \left[\ln \frac{1+\beta}{1-\beta} - i\pi \right]^2 \quad (3)$$

for $s > 4M_W^2$. Thus the contribution to $\gamma\gamma \rightarrow \chi^+\chi^-$ from rescattering of the χ is

$$\frac{e^4\pi m_H^2}{32\sin^2\theta_W(2\pi)^4} \left[\frac{m_H^2}{M_W^2} + \frac{m_H^2}{s} I \right] \frac{1}{s - m_H^2 + im_H\Gamma}. \quad (4)$$

If we take Γ in (4) to be only the $H \rightarrow \chi\chi$ contribution

$$\Gamma_{H\rightarrow\chi\chi} = \frac{e^2 m_H^3}{64\pi M_W^2 \sin^2\theta_W} \left[1 - \frac{4M_W^2}{m_H^2} \right]^{1/2} \quad (5)$$

then the contribution to the $J = 0$ partial wave amplitude from the imaginary part of I exactly cancels a_{++}^{Born} given by (1). Hence the dip of curve-b as shown in Figure 1. But even in this idealized example we should consider as well the real part of the square bracket in (4). This is shown as the curve-c in Fig. 1, where there is a dip – peak structure – the Higgs resonance asserts itself at energies slightly above the Higgs mass. The dip is not an exact zero if we use (5); however if we replace $m_H\Gamma$ in (4) by an energy dependent expression $\sqrt{s}\Gamma(s)$, the net effect of which is simply to replace m_H^2 by s only in the radical of (5), the dip is an exact zero. Its position can be calculated from the equations above - surprisingly it has only a very weak dependence on m_H ; for m_H equal 400, 600, 800 and 1000 GeV the zero occurs at \sqrt{s} of 390, 510, 570 and 560 GeV which means there are very broad valleys for the larger m_H . Because our results are only strictly correct at \sqrt{s} near m_H , we will ignore energy dependence in the widths in the remainder of this paper.

A more realistic example is to consider the scattering into longitudinal W s, $a_{++;LL}$, where the Born amplitude is still given by (1). But, of course, we must allow all polarizations of the W s in the loop integrals connecting $\gamma\gamma$ to H . This has the

effect of replacing the square bracket in (2) and (4) by²

$$6 + \frac{m_H^2}{M_W^2} + \left(\frac{m_H^2}{s} + \frac{6M_W^2}{s} - 4 \right) I. \quad (6)$$

Also we must use the full H width in the denominator of (4), $\Gamma_{H \rightarrow WW} + \Gamma_{H \rightarrow ZZ}$ (we are ignoring the top quark at this point in the discussion) and multiply (4) by $-s(1+\beta^2)/2m_H^2$ for the longitudinal polarization vectors. This is shown as the curve-b in Fig. 2. Although there is a vague resemblance in the overall dip-peak structures for the $\gamma\gamma \rightarrow \chi\chi$ the dip has disappeared completely in $\gamma\gamma \rightarrow W_L W_L$.

Now include the top quark by including it in the Higg's width

$$\Gamma_{H \rightarrow t\bar{t}} = \frac{e^2 m_t^2 m_H N_c}{32\pi M_W^2 \sin^2 \theta_W} \left[1 - \frac{4m_t^2}{m_H^2} \right]^{3/2} \quad (7)$$

and including it in the loop which connects $\gamma\gamma$ to H . This means replacing the square bracket in (2) and (4) not just by (6) but by (6) plus

$$-4e_t^2 N_c \frac{m_t^2}{M_W^2} \left[1 + \left(\frac{m_t^2}{s} - \frac{1}{4} \right) I_t \right] \quad (8)$$

where the charge of the top quark is $e_t = 2/3$, the number of colors, N_c , is 3 and I_t is given by (3) with β replaced by $\beta_t = \left(1 - \frac{m_t^2}{E^2} \right)^{1/2}$ if $s > 4m_t^2$ or, if $s < 4m_t^2$,

$$I_t = -4 \left[\sin^{-1} \left(\frac{E}{m_t} \right) \right]^2. \quad (9)$$

The effect of the top is shown in Fig. 3. The dip has reappeared for all m_t values as long as m_H is not too large.[6]

Notice that beyond the tree level, diagrams which have been included are only those which involve one-triangular loop followed by the rescattering through the exchange of the direct channel Higgs resonance. This is a reasonable approximation near

²In Ref. 6, the interference effects between resonant and nonresonant amplitude for $\gamma\gamma \rightarrow W^+ W^-$ process is also investigated. Our expressions of this process agree with their expressions at resonance energy. But there are differences at off-shell values due to the fact that some m_H^2 factors in our expressions are replaced by s in theirs. For an off-shell Higgs the expression for the loop given in Eq. (6) is not unique. These differences presumably reflect a different choice of gauge. We work in the R_ξ gauge where the couplings of the Goldstone bosons depend on m_H ; in the unitary gauge, for example, m_H does not appear.

the resonance region, where the resonance contribution is dominant. On the other hand, in the two “tail” regions of the resonance, where the resonance contribution is no longer dominating, there are competing diagrams which have been omitted and our predictions are expected to be less reliable.

For transverse Ws the Born term is slightly different than (1)

$$a_{++;++}^{\text{Born}, J=0} = \frac{e^2}{16\pi} \frac{(1+\beta)^2}{\beta} \ln \frac{1+\beta}{1-\beta} \quad (10)$$

$$a_{++;--}^{\text{Born}, J=0} = \frac{e^2}{16\pi} \frac{(1-\beta)^2}{\beta} \ln \frac{1+\beta}{1-\beta}. \quad (11)$$

Thus the relative magnitudes of the $J = 0$ Born terms for $++ : LL : --$ polarized final W 's are $1 : \left(\frac{1-\beta}{1+\beta}\right) : \left(\frac{1-\beta}{1+\beta}\right)^2$ while the final state Higgs interaction for these polarizations goes as $1 : \frac{s(1+\beta^2)}{4M_W^2} : 1$. For the $++$ final state, the ratio of the Born term to the loop contribution compared to the LL final state case, is enhanced by a factor of $\left(\frac{1+\beta}{1-\beta}\right) \cdot \frac{s(1+\beta^2)}{4M_W^2}$. This factor is about 260 for $s = (400\text{GeV})^2$. Thus the Born term for the $++$ polarization dominates over the corresponding loop contribution. No effect of Higgs interactions can be seen. Conversely the $--$ amplitude is very small as shown in Fig. 4. Notice that the coordinate is in the logarithmic scale here. It shows a full dip-peak structure with the exact behavior depending on the values of m_t , which are taken to be 140, 170 and 200 GeV for curves (a), (b) and (c) respectively.

Consider now the production of top quarks by photons or gluons, $\gamma\gamma \rightarrow t\bar{t}$ and $gg \rightarrow t\bar{t}$. Again we can have a final state interaction where the photons or gluons couple to the Higgs through a loop and the Higgs couples directly to the final tops. For photons the $J = 0$ Born terms for the two independent helicity amplitudes are given by

$$a_{++;\frac{1}{2}\frac{1}{2}}^{\text{Born}, J=0} = \frac{e^2 e_t^2}{16\pi} (1-\beta^2)^{1/2} \frac{(1+\beta)}{\beta} \ln \frac{1+\beta}{1-\beta} \quad (12)$$

$$a_{++;-\frac{1}{2}-\frac{1}{2}}^{\text{Born}, J=0} = -\frac{e^2 e_t^2}{16\pi} (1-\beta^2)^{1/2} \frac{1-\beta}{\beta} \ln \frac{1+\beta}{1-\beta}. \quad (13)$$

where β was called β_t above. For gluons of equal color the $J = 0$ Born terms are also given by (12) and (13) except for some different coupling factors. The Higgs interaction

is different in the two cases because we can have different particles in the loop. For photons it is given by (4) with (6) plus (8) in the square bracket; for gluons only (8) is in the square bracket (in order to compare the effect of different contributions to the loop we will ignore again the different gluon coupling factors). For both photons and gluons Eq. (4) must also be multiplied by

$$2m_t E \beta_t / m_H^2$$

for the spinors of the final tops.

The results of $|a_{++;\frac{1}{2}\frac{1}{2}}|^2$ for top mass of 140, 170, or 200 GeV are shown in Figs. 5a, 5b and 5c respectively. In each figure the Higgs mass is taken to be 500, 600, or 800 GeV. The gg process has big dips with small peaks (curves b, d and f), while the $\gamma\gamma$ process has the reverse (curves a, c and e). Because (12) and (13) have opposite sign this is reversed for $|a_{++;-\frac{1}{2}-\frac{1}{2}}|^2$ as shown in Fig. 6. Here the photon process has a small dip (curves a and c) which becomes a peak if m_H is large (e.g. curve e). The gluon process has a large peak (curves b, d and f). Since the $\frac{1}{2}\frac{1}{2}$ amplitude is much larger than the $-\frac{1}{2}-\frac{1}{2}$ amplitude the total cross section for $gg \rightarrow t\bar{t}$ will have a dip, except for m_H very near threshold[7]. Similar results hold for other top masses.

A final word of caution is in order here. We have discussed the behavior of the square of the J=0 partial wave amplitude for various processes. As alluded to earlier, the partial wave amplitude behavior gives the most prominent display of the interference effect. On the other hand, at the cross section level, due to the contribution of the Born term to other partial waves, the magnitude of the interference effect compared to that at the partial wave amplitude level is expected to be greatly reduced. We refer the reader to, for examples Refs 4, 6 and 7, for typical suppressions involved. Reference 7 seems to show that the structure is potentially observable at CERN Large Hadron Collider.

In conclusion we find that near the Higgs resonance region, the dip structure which appears in one piece of the Goldstone boson production contribution shows up as more complex dip-peak patterns in the full partial wave amplitude and various different final helicity states and for different intermediate state contributions. Photon- and gluon- $t\bar{t}$

production have opposite dip-peak structure with the gluon process having mostly dips for the dominant helicity state.

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Figure Captions

Figure 1. $|a_{++;LL}|^2$ vs. E for $\gamma\gamma \rightarrow$ Goldstone bosons, where E is the center of mass energy of one photon. The Higgs mass is taken to be 600 GeV. The solid line (a) is the Born contribution. The short-long dashed line (b) is $\gamma\gamma \rightarrow \chi^+\chi^-$ including only the imaginary part of the χ loop. The short dashed line (c) includes all of the χ loop.

Figure 2. $|a_{++;LL}|^2$ vs. E for $\gamma\gamma \rightarrow W_L W_L$ ignoring top quarks. m_H equals 600 GeV. The solid and short dashed lines, (a) and (c) are as in Fig. 1 for the $\gamma\gamma \rightarrow \chi^+\chi^-$ process. The short-long dashed line (b) is the process $\gamma\gamma \rightarrow W^*W^* \rightarrow H \rightarrow W_L W_L$ including all polarizations of W s in the loop connecting the photons to the Higgs and the full Higgs width in the propagator (neglecting top quarks).

Figure 3. $|a_{++;LL}|^2$ vs. E for the physical process $\gamma\gamma \rightarrow W_L W_L$ including the total gauge contribution (Fig. 2) plus the top quark contribution. The solid line is the Born term as in Fig. 1. The short-long dashed line (a), short dashed line (b), and long dashed line (c) are for $m_t = 140, 170$, and 200 GeV. Fig. 3a has $m_H = 500$ GeV, Fig. 3b has $m_H = 600$ GeV and Fig. 3c has $m_H = 800$ GeV.

Figure 4. $|a_{++;-}|^2$ vs. E for $\gamma\gamma \rightarrow W_T W_T$ the Higgs Mass is 500 GeV. The solid line is the Born amplitude. The other lines, (a), (b), (c), include the full gauge contribution to the final state interaction as well as a top quark of mass 140, 170, and 200 GeV as in Fig. 3.

Figure 5. $|a_{++;\frac{1}{2}\frac{1}{2}}|^2$ vs. E for $\gamma\gamma \rightarrow t\bar{t}$ or $gg \rightarrow t\bar{t}$. The solid line is the Born term which has been normalized to be the same for the gluon and photon processes. Fig. 5a has $m_t = 140$ GeV, Fig. 5b has $m_t = 170$ GeV and Fig. 5c has $m_t = 200$ GeV. In each figure the Higgs mass is taken to be 500, 600, and 800 GeV, the corresponding curves (a), (c) and (e) are for the photon process, while the corresponding curves (b), (d) and (f) are for the gluon process.

Figure 6. $|a_{++;-\frac{1}{2}-\frac{1}{2}}|^2$ vs. E for $m_t = 170$ GeV. Otherwise the same as Fig. 5.

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